15393

M. Tech 1st Semester Examination

Advanced Mathematics (NS)

EE1-513

Time : 3 Hours   Max. Marks : 100

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt any Five Questions. All questions carry equal marks. Use of non-programmable calculator is allowed.

1. (a) Evaluate Laplace transform of the function

$$f(t) = e^{-1}\sinh(2t) + t^2 e^{3t} + \frac{\sin(t)}{t}$$

   (b) Evaluate inverse Laplace transform of the function
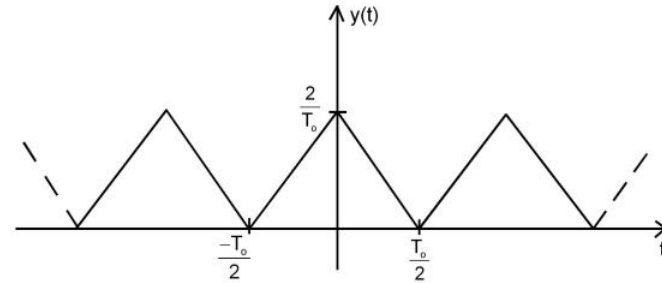
$$F(s) = \frac{3s+1}{(s-1)\,(s^2+1)}.$$ (20)

2. (a) Determine Z-transform of the sequence $\{x_k\} = \left\{\left(\frac{1}{2}\right)^k\right\}$,

   Using this result obtain the Z-transform of the sequence

$$\left\{k\left(\frac{1}{2}\right)^k\right\}.$$

   (b) Find inverse Z-transform of $Y(z) = \dfrac{z}{\left(z^2 + a^2\right)}$ (20)

3. (a) Determine Fourier Transform of $e^{-\alpha t^2} * e^{-\beta t^2}$

   (b) Determine the Fourier series of the periodic function defined in the following figure   (20)

4. (a) Compute the discrete Fourier transform of x(k - 3). Compare results with those obtained from the time-shifting relationship.

   (b) Discuss the discrete correlation theorem.   (20)

5. (a) Derive the FFT algorithm for $N = r_1 r_2$ for the case where the components of n are separated.

   (b) Find the two-dimensional Fourier transform of the function

$$h(x,y) = \begin{cases} 1 & -1 < x < 1, \quad -1 < y < 1 \\ 0 & , \qquad\qquad\qquad \text{otherwise} \end{cases}$$ (20)

6. (a) Derive the differential equation for the RC circuit. Find charge in the case when E(t)=E(constant) and q(0)=q$_0$. Plot for q(t) and determine when the charge has its maximum and minimum value.

   (b) Find the solution of the system of linear differential equation

$$x_1' = 3x_1 + 3x_2 + 8, \quad x_2' = x_1 + 5x_2 + 4e^{3t},$$ (20)

7. (a) Find solution of the difference equation $8y_{k+2} - 6y_{k+1} + y_k$

   $= 9$ $(k \geq 0)$ given that $y_0 = 1$ and $y_1 = \dfrac{3}{2}$

   (b) Find phase portrait of the system $x' = -2y - x\sin(xy)$, $y' = 2x + y\sin(xy)$.   (20)

8. (a) Define discrete random variable. A random variable X has the following probability function :

Values of X:	-2	-1	0	1	2	3
P(X):	0.1	k	0.2	2k	0.3	k

   find the value of k, $P(X<0)$ and $P(-2 <X< 2)$. Also determine the distribution function of X.

   (b) Determine the Binomial distribution for which the mean is 4 and variance 3.

   (c) Find the probability that in a family of 4 children there will be (i) atleast 1 boy, (ii) at least 1 boy and atleast 1 girl. Assuming that the probability of a male birth is 0.5.   (20)